CE 6402 STRENGTH OF MATERIAL
(REGULATION-2013)

UNIT – I ENERGY PRINCIPLES

1.Define: Strain Energy

When an elastic body is under the action of external forces the body deforms and work is done by these forces. If a strained, perfectly elastic body is allowed to recover slowly to its unstrained state. It is capable of giving back all the work done by these external forces. This work done in straining such a body may be regarded as energy stored in a body and is called strain energy or resilience.

2. Define: Proof Resilience.

The maximum energy stored in the body within the elastic limit is called Proof Resilience.

3. Write the formula to calculate the strain energy due to axial loads (tension).

$$U = \int \frac{P^2}{2AE} dx \qquad \text{limit } 0 \text{ to } L$$

Where,

P = Applied tensile load.
L = Length of the member
A = Area of the member
E = Young's modulus.

4. Write the formula to calculate the strain energy due to bending.

$$U = \int \frac{M^2 dx}{2EI} \qquad \text{limit } 0 \text{ to } L$$

Where,

M = Bending moment due to applied loads.
E = Young's modulus
I = Moment of inertia

5. Write the formula to calculate the strain energy due to torsion

$$U = \int \frac{T^2 dx}{2GJ} \qquad \text{limit } 0 \text{ to } L$$

Where, T = Applied Torsion
G = Shear modulus or Modulus of rigidity
J = Polar moment of inertia

6.Write the formula to calculate the strain energy due to pure shear

$$U = K \int \frac{V^2 dx}{2GA} \qquad \text{limit } 0 \text{ to } L$$

Where, V= Shear load
G = Shear modulus or Modulus of rigidity
A = Area of cross section.
K = Constant depends upon shape of cross section.

7. Write down the formula to calculate the strain energy due to pure shear, if shear stress is given.

$$U = \frac{\tau^2 V}{2G}$$

Where, τ = Shear Stress
G = Shear modulus or Modulus of rigidity
V = Volume of the material.

8. Write down the formula to calculate the strain energy , if the moment value is given

$$U = \frac{M^2 L}{2EI}$$

Where, M = Bending moment
L = Length of the beam
E = Young's modulus
I = Moment of inertia

9. Write down the formula to calculate the strain energy , if the torsion moment value is given.

$$U = \frac{T^2 L}{2GJ}$$

Where, T = Applied Torsion
L = Length of the beam
G = Shear modulus or Modulus of rigidity
J = Polar moment of inertia

10. Write down the formula to calculate the strain energy, if the applied tension load is given.

$$U = \frac{P^2 L}{2AE}$$

Where,
P = Applied tensile load.
L = Length of the member
A = Area of the member
E = Young's modulus.

11. Write the Castigliano's first theorem.

In any beam or truss subjected to any load system, the deflection at any point is given by the partial differential coefficient of the total strain energy stored with respect to force acting at a point.

$$\delta = \frac{\partial U}{\partial P}$$

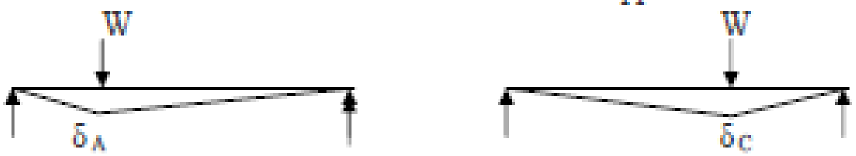
Where,
δ = Deflection
U= Strain Energy stored
P = Load

12. What are uses of Castigliano's first theorem?

1. To determine the deflection of complicated structure.
2. To determine the deflection of curved beams springs.

13. Define: Maxwell Reciprocal Theorem.

In any beam or truss the deflection at any point 'A' due to a load 'W' at any other point 'C' is the same as the deflection at 'C' due to the same load 'W' applied at 'A'.



$\delta_A = \delta_C$

14. Define: Unit load method.

The external load is removed and the unit load is applied at the point, where the deflection or rotation is to found.

15. Give the procedure for unit load method.

1. Find the forces P1, P2, ……. in all the members due to external loads.
2. Remove the external loads and apply the unit vertical point load at the joint if the vertical deflection is required and find the stress.
3. Apply the equation for vertical and horizontal deflection.

16. Compare the unit load method and Castigliano's first theorem

In the unit load method, one has to analyze the frame twice to find the load and deflection. While in the latter method, only one analysis is needed.

17. Find the strain energy per unit volume, the shear stress for a material is given as 50 N/mm ^{2}. Take G= 80000 N/mm ^{2}.

$U = \dfrac{\tau^2}{2G}$ per unit volume

$= 50^2 / (2 \times 80000)$

$= 0.015625$ N/mm ^{2}. per unit volume.

18. Find the strain energy per unit volume, the tensile stress for a material is given as 150 N/mm ^{2}. Take E = 2 x10 N/mm ^{2}.

$U = \dfrac{f^2}{2E}$ per unit volume

$= (150)^2 / (2 \times (2 \times 10^2))$

$= 0.05625$ N/mm ^{2}. per unit volume.

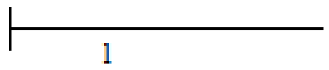
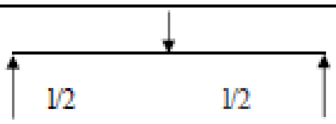
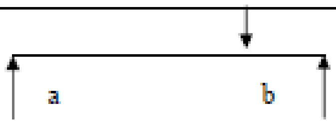
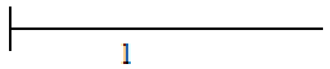
19.Define : Modulus of resilience.

The proof resilience of a body per unit volume. (ie) The maximum energy stored in the body within the elastic limit per unit volume.

20. Define : Trussed Beam.

A beam strengthened by providing ties and struts is known as Trussed Beams.

21. Deflection of beams

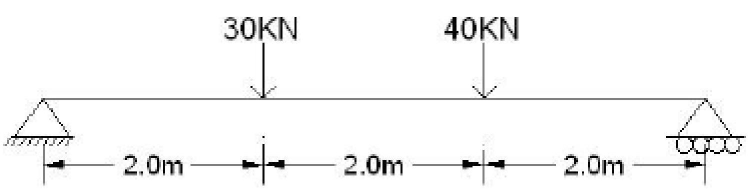
Type of beam	Deflection
	$\delta = wl^3 / 3EI$
	$\delta = wl^3 / 48EI$
	$\delta = wa^2b^2 / 3EIl$
	$\delta = 5wl^4 / 384EI$
	$\delta = wl^4 / 8EI$
	$\delta = \Pi wr^3$

PART – B (16 Marks)

1. For the beam shown in Fig, find the deflection at C and slope at D **(AUC Apr/May 2010)**

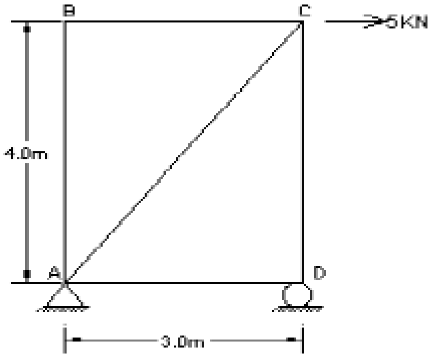
$I = 40 \times 10^7\ mm^4$

E = 200 GPa.



2. For the truss shown in fig find the horizontal movement of the roller at D AB, BC, CD area = 8 cm^2

AD and AC = 16 cm^2 , E= 2x10^5 N/mm^2



3. Derive the expression for strain energy in torsion of a circular shaft of length ll and radius R subjected to a torque Tproducing a twist Ө in the length of the shaft for the following casas

   i) solid circular shaft and

   ii) hollow circular shaft, with an external radius R and internal radius r

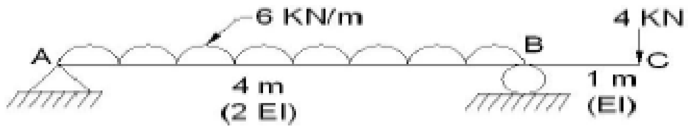
4. i) An axial pull of 40 kN is suddenly applied to a steei rod 2m long and 1000mm^2 in cross section . calculate the strain energy that can be absorbed if E = 200 GN/m^2.

   ii) a cantilever of rectangular section breadth b, depth d and of length l carries uniformly distributed looad spread from free end to the mid section of the cantilever. Using castiglianos theorem find : slope and deflection due to bending at the free end.

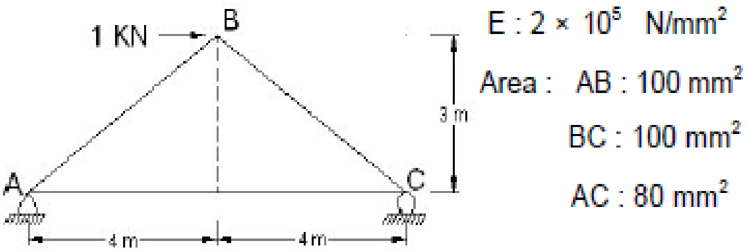
5. a beam 4m in length is simply supported at the ends and carries a uniformly distributed load of 6 kN/m. determine the strain energy stored in the beam. Take E= 200Gpa and I=1440 cm^2.

6. a beam simply supported over a span of 3m carries a UDL of 20 kN/m over the entire span. The flexural rigidity EI=2.25 MNm^2 using castiglianos theorem determine. The deflection at the centre of the beam

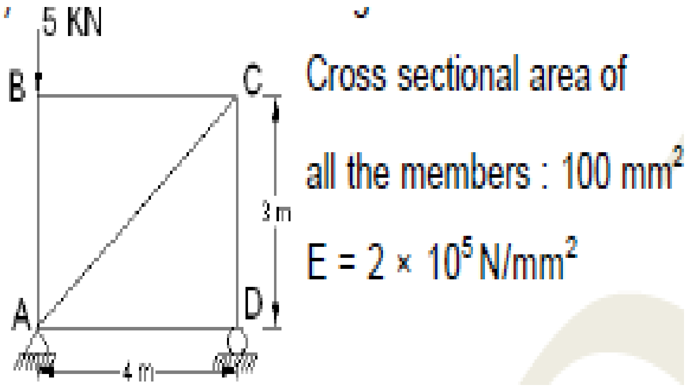
7. For the beam shown in fig. find the slope and deflection at 'C'. **(AUC Nov/Dec 2011)**



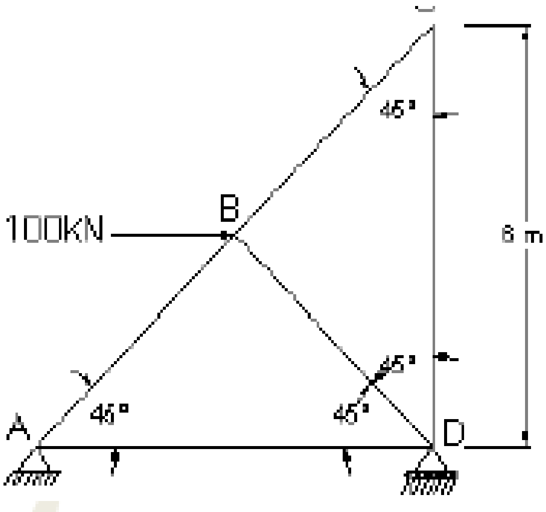
8. i) For the truss shown in fig. find the total strain energy stored. **(AUC Nov/Dec 2011)**



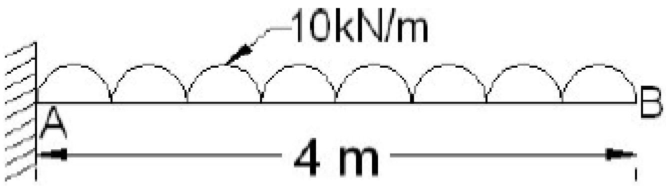
ii) for the truss shown in fig find the vertical deflection at C



9. Determine vertical and horizontal deflection of joints c as shown in fig using principle of virtual work. take $E=200KN/mm^2$ and $A=600mm^2$ for all the members



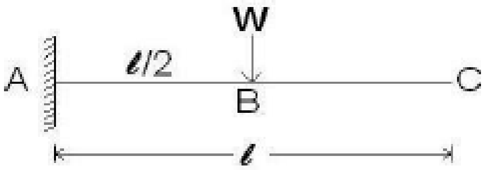
10. using castiglianos theorem find the slope and deflection at B for the cantilever beam shown in fig take $E= 2x10^5$ N/mm^2 and $I = 1x10^8$ mm^4



11. i. Derive a relation for strain energy due to shear force. **(4m)**

ii. Derive a relation for maximum deflection of a simply supported beam with uniformly distributed load over entire span. Use strain energy method. **(12m)**

12. Determine the deflection at C of the beam given in Fig. Use principal of virtual work.



13. The external diameter of a hollow shaft is twice the internal diameter. It is subjected to pure torque and it attains a maximum shear stress 'τ'. Show that the strain energy stored

Per unit volume of the shaft is $\frac{5\tau^2}{16C}$ such a shaft is required to transmit 5400kw at 110 r.p.m with uniform torque , the maximum stress not exceeding 84 MN/m^2 . determine

i. The shaft diameters
ii. The strain energy stored per m^3 take C= 90GN/m^2.

UNIT -2 INDETERMINATE BEAM

Part- A

1. Explain with examples the statically indeterminate structures.

If the forces on the members of a structure cannot be determined by using conditions of equilibrium ($\sum Fx = 0$, $\sum Fy = 0$, $\sum M = 0$), it is called statically indeterminate structures.

Example: Fixed beam, continuous beam.

2. Differentiate the statically determinate structures and statically indeterminate structures?

Sl.No	statically determinate structures	statically indeterminate structures
1.	Conditions of equilibrium are sufficient to analyze the structure	Conditions of equilibrium are insufficient to analyze the structure
2.	Bending moment and shear force is independent of material and cross sectional area.	Bending moment and shear force is dependent of material and independent of cross sectional area.
3.	No stresses are caused due to temperature change and lack of fit.	Stresses are caused due to temperature change and lack of fit.

3. Define: Continuous beam.

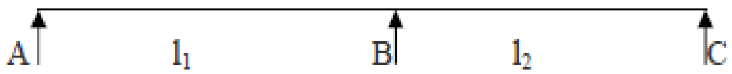
A Continuous beam is one, which is supported on more than two supports. For usual loading on the beam hogging (- ive) moments causing convexity upwards at the supports and sagging (+ ve) moments causing concavity upwards occur at mid span.

4. What are the advantages of Continuous beam over simply supported beam?

1. The maximum bending moment in case of continuous beam is much less than in case of simply supported beam of same span carrying same loads.

2. In case of continuous beam, the average bending moment is lesser and hence lighter materials of construction can be used to resist the bending moment.

5. Write down the general form of Clapeyron's three moment equations for the continuous beam.



$$M_a l_1 + 2 M_b (l_1 + l_2) + M_c l_2 = -\left(\frac{6 A_1 \bar{x}_1}{l_1} + \frac{6 A_2 \bar{x}_2}{l_2} \right)$$

where,

M_a = Hogging bending moment at A

M_b = Hogging bending moment at B

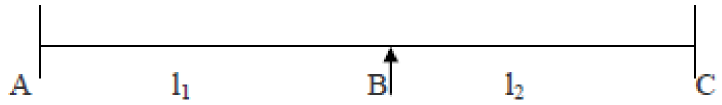
M_c = Hogging bending moment at C

l_1 = length of span between supports A,B

l_2 = length of span between supports B, C

$\bar{x}_1$ = CG of bending moment diagram from support A
$\bar{x}_2$ = CG of bending moment diagram from support C
A_1 = Area of bending moment diagram between supports A,B
A_2 = Area of bending moment diagram between supports B, C

6. Write down the Clapeyron's three moment equations for the continuous beam with sinking at the supports.

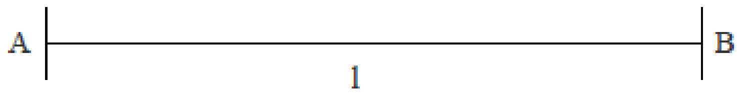


$$M_a l_1 + 2M_b(l_1 + l_2) + M_c l_2 = -\left(\frac{6A_1\bar{x}_1}{l_1} + \frac{6A_2\bar{x}_2}{l_2}\right) + 6EI\left(\frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}\right)$$

where,

M_a = Hogging bending moment at A
M_b = Hogging bending moment at B
M_c = Hogging bending moment at C
l_1 = length of span between supports A,B
l_2 = length of span between supports B, C
$\bar{x}_1$ = CG of bending moment diagram from support A
$\bar{x}_2$ = CG of bending moment diagram from support C
A_1 = Area of bending moment diagram between supports A,B
A_2 = Area of bending moment diagram between supports B, C
δ_1 = Sinking at support A with compare to sinking at support B
δ_2 = Sinking at support C with compare to sinking at support B

7. Write down the Clapeyron's three moment equations for the fixed beam

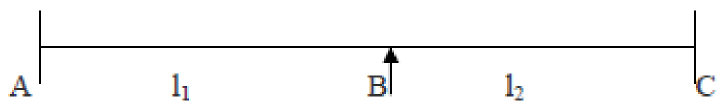


$$M_a + 2M_b = \left(\frac{6A\bar{x}}{l^2}\right)$$

where,

M_a = Hogging bending moment at A
M_b = Hogging bending moment at B
l = length of span between supports A,B
$\bar{x}$ = CG of bending moment diagram from support A
A = Area of bending moment diagram between supports A,B

8. Write down the Clapeyron's three moment equations for the continuous beam carrying UDL on both the spans.



$$M_a l_1 + 2M_b l_2 + M_c l_2 = \left(\frac{6A_1\bar{x}_1}{l_1} + \frac{6A_2\bar{x}_2}{l_2}\right) = \frac{w_1 l_1^3}{4} + \frac{w_2 l_2^3}{4}$$

where,

M_a = Hogging bending moment at A
M_b = Hogging bending moment at B
M_c = Hogging bending moment at C
l_1 = length of span between supports A,B
l_2 = length of span between supports B, C

9. Give the values of ($6A_1 x_1 / l_1$), ($6A_2 x_2 / l_2$) values for different type of loading.

Type of loading	$6A_1 x_1 / l_1$	$6A_2 x_2 / l_2$
UDL for entire span	$wl^3 / 4$	$wl^3 / 4$
Central point loading	$(3/8)\ Wl^2$	$(3/8)\ Wl^2$
Uneven point loading	$(wa / l) / (l^2 - a^2)$	$(wb / l) / (l^2 - b^2)$

10. Give the procedure for analyzing the continuous beams with fixed ends using three moment equations?

The three moment equations, for the fixed end of the beam, can be modified by imagining a span of length l_0 and moment of inertia, beyond the support the and applying the theorem of three moments as usual.

11. Define Flexural Rigidity of Beams.

The product of young's modulus (E) and moment of inertia (I) is called Flexural Rigidity (EI) of Beams. The unit is N mm^2.

12. What is a fixed beam?

A beam whose both ends are fixed is known as a fixed beam. Fixed beam is also called as built-in or encaster beam. Incase of fixed beam both its ends are rigidly fixed and the slope and deflection at the fixed ends are zero.

13. What are the advantages of fixed beams?

(i) For the same loading, the maximum deflection of a fixed beam is less than that of a simply supported beam.
(ii) For the same loading, the fixed beam is subjected to lesser maximum bending moment.
(iii) The slope at both ends of a fixed beam is zero.
(iv) The beam is more stable and stronger.

14. What are the disadvantages of a fixed beam?

(i) Large stresses are set up by temperature changes.
(ii) Special care has to be taken in aligning supports accurately at the same lavel.
(iii) Large stresses are set if a little sinking of one support takes place.
(iv) Frequent fluctuations in loadingrender the degree of fixity at the ends very uncertain.

15. Write the formula for deflection of a fixed beam with point load at centre.

$$\delta = -\frac{wl^3}{192\,EI}$$

This defection is ¼ times the deflection of a simply supported beam.

16. Write the formula for deflection of a fixed beam with uniformly distributed load..

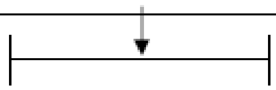
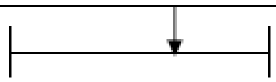
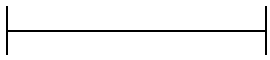
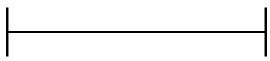
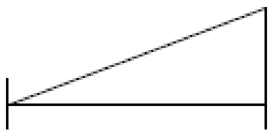
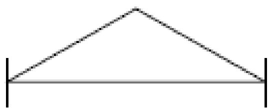
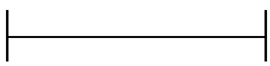
$$\delta = -\frac{wl^4}{384\,EI}$$

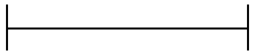
This defection is 5 times the deflection of a simply supported beam.

17. Write the formula for deflection of a fixed beam with eccentric point load..

$$\delta = -\frac{wa^3b^3}{3\,EI\,l^3}$$

18. What are the **fixed end moments** for a **fixed beam** with the given loading conditions.

Type of loading	M_{AB}	M_{BA}
	$-wl / 8$	$wl / 8$
	$-wab^2 / l^2$	wab^2 / l^2
	$-wl^2 / 12$	$wl^2 / 12$
	$\frac{-wa^2}{12\,l^2}(6l^2 - 8la + 3a^2)$	$\frac{-wa^2}{12\,l^2}(4l - 3a)$
	$-wl^2 / 30$	$-wl^2 / 30$
	$\frac{-5}{96}\,wl^2$	$\frac{-5}{96}\,wl^2$
	$M / 4$	$M / 4$

	$\frac{M_b\,(3a - l)}{l^2}$	$\frac{M_a\,(3b - l)}{l^2}$

PART-B

1. For the fixed beam shown in Fig, draw the SFD and BMD. (AUC Apr/May 2010)

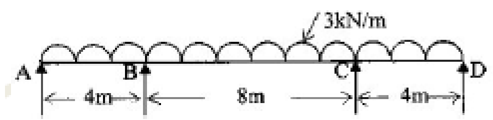
2. For the continuous beam shown in Fig, draw SFD and BMD all the supports are at same level. (AUC Apr/May 2010)

3. A fixed beam AB of 4.5m span carries a point load of 80 kN at its mid span and a uniformly distributed load of 15 kN/m throughout its entire span. Find the following:

   i. Fixing moment at the enda and
   ii. Reaction at the supports
   Also draw SF and BM diagram

4. A continuous beam ABCD of uniform cross-section is loaded as shown in Figure Find the following: (AUC Nov/Dec 2010)

   i. Bending moment at the supports
   ii. Reaction at the supports also draw BM and SFdiagram

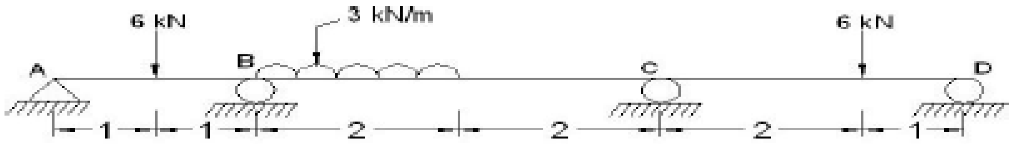


5. A FIXED beam of 6m span is loaded with point load of 150KN at distance of 2m from each support draw the BM and SF diagram also find the maximum deflection take E=200Gpa and I=8x108mm^4

6. a continuous beam consists of three successive span of 6m, 12m and 4m and carries load of 2 kN/m, 1 kN/m and 3kN/m respectively on the span draw BM and SF diagram for the continuous beam

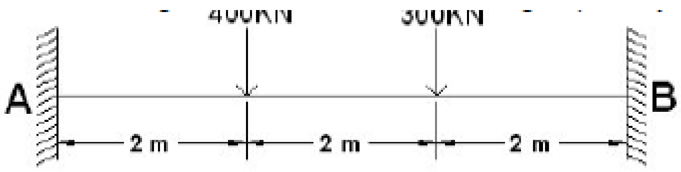
7. A fixed beam AB is 6 m span and carries a point load 10 kN at 1 m from left end. It also carries a clockwise moment at 1 m from right end,10 kN/m. Draw SFD and BMD indicating the salient points. **(AUC Nov/Dec 2011)**

8. A continuous beam ABCD in shown in Fig. Draw SFD and BMD indicating the salient points. **(AUC Nov/Dec 2011)**



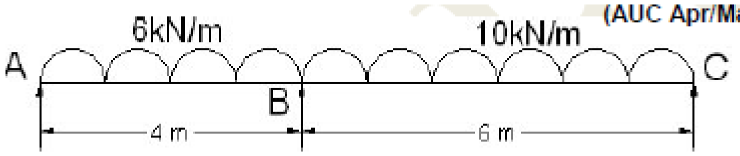
(EI constant throughout)

9. Draw the SF and BM diagram for the beam shown in the fig



10. Draw the SF and BM diagram for the beam shown in the fig. use three moment equation

(AUC Apr/May 2012)



11.

A fixed beam of 6m length is loaded with two equal point loads of 150kN each at distance of 2m from each support. Draw the BMD and SFD. $E = 2 \times 10^8$ kN/m^2, $I = 8 \times 10^8$ mm^4.

12.

A continuous beam ABC 8m long consists of two spans AB = 3m and BC = 5m. The span AB carries a load of 50 kN/m while the span BC carries a load of 10 kN/m. Find the support moments and the reactions at the supports.

UNIT-3 COLUMN AND CYLINDER

Part-A

1.Define: Column and strut.

A column is a long vertical slender bar or vertical member, subjected to an axial compressive load and fixed rigidly at both ends.

A strut is a slender bar or a member in any position other than vertical, subjected to a compressive load and fixed rigidly or hinged or pin jointed at one or both the ends.

2. What are the types of column failure?

1. Crushing failure:

The column will reach a stage, when it will be subjected to the ultimate crushing stress, beyond this the column will fail by crushing The load corresponding to the crushing stress is called crushing load. This type of failure occurs in short column.

2. Buckling failure:

This kind of failure is due to lateral deflection of the column. The load at which the column just buckles is called buckling load or crippling load or critical load. This type of failure occurs in long column.

3. What is slenderness ratio (buckling factor)? What is its relevance in column?

It is the ratio of effective length of column to the least radius of gyration of the cross sectional ends of the column.

Slenderness ratio = l eff / r

l eff = effective length of column

r = least radius of gyration

Slenderness ratio is used to differentiate the type of column. Strength of the column depends upon the slenderness ratio, it is increased the compressive strength of the column decrease as the tendency to buckle is increased.

4. What are the factors affect the strength column?

1.Slenderness ratio

Strength of the column depends upon the slenderness ratio, it is increased the compressive strength of the column decrease as the tendency to buckle is increased.

2. End conditions: Strength of the column depends upon the end conditions also.

5. Differentiate short and long column

Short column	Long column
1. It is subjected to direct compressive stresses only.	It is subjected to buckling stress only.
2. Failure occurs purely due to crushing only.	Failure occurs purely due to bucking only.
3. Slenderness ratio is less than 80	Slenderness ratio is more than 120.

4.It's length to least lateral dimension is less than 8. (L / D ‹ 8)	It's length to least lateral dimension is more than 30. (L / D › 30)

6. What are the assumptions followed in Euler's equation?

1. The material of the column is homogeneous, isotropic and elastic.
2. The section of the column is uniform throughout.
3. The column is initially straight and load axially.
4. The effect of the direct axial stress is neglected.
5. The column fails by buckling only.

7. What are the limitations of the Euler's formula?

1. It is not valid for mild steel column. The slenderness ratio of mild steel column is less than 80.

2. It does not take the direct stress. But in excess of load it can withstand under direct compression only.

8. Write the Euler's formula for different end conditions.

1. Both ends fixed.

$$P_E = \frac{\pi^2 EI}{(0.5L)^2}$$

2. Both ends hinged

$$P_E = \frac{\pi^2 EI}{(L)^2}$$

3. One end fixed ,other end hinged.

$$P_E = \frac{\pi^2 EI}{(0.7L)^2}$$

4. One end fixed, other end free.

$$P_E = \frac{\pi^2 EI}{(2L)^2}$$

L = Length of the column

9. Define: Equivalent length of the column.

The distance between adjacent points of inflection is called equivalent length of the column. A point of inflection is found at every column end, that is free to rotate and every point where there is a change of the axis. ie, there is no moment in the inflection points. (Or)

The equivalent length of the given column with given end conditions, is the length of an equivalent column of the same material and cross section with hinged ends , and having the value of the crippling load equal to that of the given column.

10. What are the uses of south well plot? (column curve).

The relation between the buckling load and slenderness ratio of various column is known as south well plot.

The south well plot is clearly shows the decreases in buckling load increases in slenderness ratio.

It gives the exact value of slenderness ratio of column subjected to a particular amount of buckling load.

11. Give Rakine's formula and its advantages.

$$P_R = \frac{f_C A}{(1 + a (l_{eff} / r)^2)}$$

where, P_R = Rakine's critical load

f_C = yield stress

A = cross sectional area

a = Rakine's constant

l_{eff} = effective length

r = radius of gyration

In case of short column or strut, Euler's load will be very large. Therefore, Euler's formula is not valid for short column. To avoid this limitation, Rankine's formula is designed. The Rankine's formula is applicable for both long and short column.

12. Write Euler's formula for maximum stress for a initially bent column?

$$\sigma\ max = P / A + (M\ max / Z)$$

$$= P / A + \frac{P\ a}{(1 - (P / P_E))Z}$$

Where, P = axial load

A = cross section area

P_E = Euler's load

a = constant

Z = section modulus

13. Write Euler's formula for maximum stress for a eccentrically loaded column?

$$\sigma\ max = P / A + (M\ max / Z)$$

$$= P / A + \frac{P\ e\ Sec\ (l\ eff / 2)\ \sqrt{(P/EI)}}{(1 - (P / P_E))\ Z}$$

Where, P = axial load

A = cross section area

P_E = Euler's load

e = eccentricity

Z = section modulus

EI = flexural rigidity

14. What is beam column? Give examples.

Column having transverse load in addition to the axial compressive load are termed as beam column.

Eg : Engine shaft, Wing of an aircraft.

15. Define buckling factor and buckling load.

Buckling factor : It is the ratio between the equivalent length of the column to the minimum radius of gyration.

Buckling load : The maximum limiting load at which the column tends to have lateral displacement or tends to buckle is called buckling or crippling load. The buckling takes place about the axis having minimum radius of gyration, or least moment of inertia.

16. Define safe load.

It is the load to which a column is actually subjected to and is well below the buckling load. It is obtained by dividing the buckling load by a suitable factor of safety (F.O.S).

$$\text{Safe load} = \frac{\text{Buckling load}}{\text{Factor of safety}}$$

17. Write the general expressions for the maximum bending moment, if the deflection curve equation is given.

$$BM = - EI\,(d^2y / dx^2)$$

18. Define thick cylinders.

Thick cylinders are the cylindrical vessels, containing fluid under pressure and whose wall thickness is not small. ($t \geq d/20$)

19. State the assumptions made in Lame's theory.

i) The material is homogeneous and isotropic.
ii) Plane sections perpendicular to the longitudinal axis of the cylinder remain plane after the application of internal pressure.
iii) The material is stressed within the elastic limit.
iv) All the fibres of the material are to expand or contract independently without being constrained by the adjacent fibres.

20. Write Lame's equation to find out stesses in a thick cylinder.

$$\text{Radial stress} = \sigma_r = \frac{b}{r^2} - a$$

$$\text{Circumferential or hoop stress} = \sigma_c = \frac{b}{r^2} + a$$

21. State the variation of hoop stress in a thick cylinder.

The hoop stress is maximum at the inner circumference and minimum at the outer circumference of a thick cylinder.

22. How can you reduce hoop stress in a thick cylinder.

The hoop stress in thick cylinders are reduced by shrinking one cylinder over another cylinder.

PART B

1. i) Derive the Euler's equation for column with two ends fixed. **(AUC Apr/May 2010)**

   ii) A circular bar of uniform section is loaded with a tensile load of 500 kN. The line of action of the load is off the axis of the bar by 10 mm. Determine the diameter of the rod, if permissible stress of the material of the rod is 140 N / mm^2.

2.

Find the greatest length of a mild steel rod of 30 mm × 30 mm which can be used as a compressive member with one end fixed and the other end hinged. It carries a working load of 40 kN. Factor of safety = 4, $\alpha = \frac{1}{7500}$ and $\sigma_c = 300 N/mm^2$. Compare the result

With Euer $E=2x10^5 N/mm^2$.

3.

i) What are the assumptions and limitations of Euler's theory for long columns?

ii) A slender pin ended aluminium column 2.0 m long and of circular cross section it to have an outside diameter of 50 mm. Calculate the necessary internal diameter to prevent failure by buckling if the actual load applied is l2kN and the critical load applied is twice the actual load. Take E for aluminium as 70 GN/m^2.

4.

i) Describe the Rankine's method for columns subjected to Eccentricity.

ii) From the following data of a column of circular section calculate the extreme stresses on the column section. Also find the maximum eccentricity in order that there may be no tension anywhere on the section.

External diameter = 20 cm Internal diameter = 6

cm Length of the column = 4 m

Load carried by the column = 175 kN

Eccentricity of the load = 2.5 cm (from the axis of the column)

End conditions = Both ends fixed

Young's modulus = 94 GN/m^2. **(AUC Nov/Dec 2010)**

5. A 1.5m long cast iron column has a circular cross section of 50mm diameter. One end of the column is fixed in direction and position and the other is free. Taking factor of safety as 3, calculate the safe load using Rankine-Gordon formula. Take yield stress as 560 MPa and constant α = 1/1600. **(AUC Apr/May 2011)**

6. A pipe of 200mm internal diameter and 50mm thickness carries a fluid at a pressure of 10 MPa. Calculate the maximum and minimum intensities of circumferential stress across the section. Also sketch the radial stress distribution and circumferential stress distribution across the section. **(AUC Apr/May 2011)**

7. i) A rectangular strut is 25 cm × 15 cm. It carries a load of 60 kN at an eccentricity of 2 cm in a plane bisecting the thickness. Find the minimum and maximum stresses

   Develop in the section.

   ii) derive the Euler's equation for a long column with both ends hinged

8. i) A hollow cylindrical cast iron column is 3.50 long with both ends fixed. Diameter the minimum diameter of the column if it has to carry a safe load of 300 KN with a factor of safety 4 external diameter is 1.25 times the internal diameter a= 1/1600, σ_c = 550 MN/m^2, in rankine's formula

   ii) Define thick cylinder and draw the hoop stress distribution for a solid circular cylinder.

9. Derive the expression for the buckling load of an Euler's column fixed at one end and the hinged at other end

10. A short length of a tube of 60mm external diameter and with thickness 5mm, failed in compression at a load of 250kN. When the same is tested as a strut with both ends hinged 2m long, it failed at a load of 150kN. Find the value of constant 'α' in Rankine's formula. **(AUC Apr/May 2012)**

11. Derive the expression for the crippling load when one end of the column is fixed and the other end is free.

12. calculate the euler's critical load for a strut of T section the flange width being 10cm, overall depth 8cm and both flange and stem 1cm thick. The strut is 3m long and is built in at both ends take E= $2x10^5$ N/mm^2

13. Derive Euler's crippling load for the following cases:

    i. Both ends hinged. **(8m)**

    ii. One end is fixed and other end free **(8m)**

UNIT-4 STATE OF STRESS IN THREE DIMENSIONS

Part-A

1. What are the types of failures?

   1. Brittle failure:

      Failure of a material represents direct separation of particles from each other, accompanied by considerable deformation.

   2. Ductile failure:

      Slipping of particles accompanied, by considerable plastic deformations.

2.List out different theories of failure

1. Maximum Principal Stress Theory. (Rakine's theory)
2. Maximum Principal Strain Theory. (St. Venant's theory)
3. Maximum Shear Stress Theory. (Tresca's theory or Guest's theory)
4. Maximum Shear Strain Theory. (Von –Mises- Hencky theory or Distortion energy theory)
5. Maximum Strain Energy Theory. (Beltrami Theory or Haigh's theory)

3. Define: Maximum Principal Stress Theory. (Rakine's theory)

According to this theory, the failure of the material is assumed to take place when the value of the maximum Principal Stress (σ_1) reaches a value to that of the elastic limit stress(f_y) of the material. $\sigma_1 = f_y$.

4. Define: Maximum Principal Strain Theory. (St. Venant's theory)

According to this theory, the failure of the material is assumed to take place when the value of the maximum Principal Stain (e_1) reaches a value to that of the elastic limit strain(f_y / E) of the material.

$$e_1 = f_y / E$$

In 3D, $e_1 = 1/E[\sigma_1 - (1/m)(\sigma_2 + \sigma_3)] = f_y / E \rightarrow [\sigma_1 - (1/m)(\sigma_2 + \sigma_3)] = f_y$

In 2D, $\sigma_3 = 0 \rightarrow e_1 = 1/E[\sigma_1 - (1/m)(\sigma_2)] = f_y / E \rightarrow [\sigma_1 - (1/m)(\sigma_2)] = f_y$

5. Define : Maximum Shear Stress Theory. (Tresca's theory)

According to this theory, the failure of the material is assumed to take place when the maximum shear stress equal determined from the simple tensile test.

In 3D, $(\sigma_1 - \sigma_3) / 2 = f_y /2 \rightarrow (\sigma_1 - \sigma_3) = f_y$

In 2D, $(\sigma_1 - \sigma_2) / 2 = f_y /2 \rightarrow \sigma_1 = f_y$

6. Define : Maximum Shear Strain Theory (Von –Mises- Hencky theory or Distortion energy theory)

According to this theory, the failure of the material is assumed to take place when the maximum shear strain exceeds the shear strain determined from the simple tensile test.

In 3D, shear strain energy due to distortion $U = (1/12G)[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$

Shear strain energy due to simple tension, $U = f_y^2 / 6G$

$$(1/12G)[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = f_y^2 / 6G$$

$$[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = 2 f_y^2$$

In 2D, $[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - 0)^2 + (0 - \sigma_1)^2] = 2 f_y^2$

7. Define: Maximum Strain Energy Theory (Beltrami Theory)

According to this theory, the failure of the material is assumed to take place when the maximum strain energy exceeds the strain energy determined from the simple tensile test.

In 3D, strain energy due to deformation $U = (1/2E)[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (1/m)(\sigma_1\sigma_2 + \sigma_2\sigma_2 + \sigma_2\sigma_2)]$

strain energy due to simple tension, $U = f_y^2 / 2E$

$$(1/2E)[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (2/m)(\sigma_1\sigma_2 + \sigma_2\sigma_2 + \sigma_2\sigma_2)] = f_y^2 / 2E$$

$$[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (2/m)(\sigma_1\sigma_2 + \sigma_2\sigma_2 + \sigma_2\sigma_2)] = f_y^2$$

In 2D, $[\sigma_1^2 + \sigma_2^2 - (2/m)(\sigma_1\sigma_2)] = f_y^2$

8. What are the theories used for ductile failures?
   1. Maximum Principal Strain Theory. (St. Venant's theory)
   2. Maximum Shear Stress Theory. (Tresca's theory)
   3. Maximum Shear Strain Theory. (Von –Mises- Hencky theory or Distortion energy theory)

9. Write the limitations of Maximum Principal Stress Theory. (Rakine's theory)

1. This theory disregards the effect of other principal stresses and effect of shearing stresses on other planes through the element.

2. Material in tension test piece slips along 45^0 to the axis of the test piece, where normal stress is neither maximum nor minimum, but the shear stress is maximum.

3.Failure is not a brittle, but it is a cleavage failure.

10. Write the limitations of Maximum Shear Stress Theory. (Tresca's theory).

This theory does not give the accurate results for the state of stress of pure shear in which the maximum amount of shear is developed (in torsion test).

11.Write the limitations of Maximum Shear Strain Theory.(Von –Mises- Hencky theory or Distortion energy theory).

It cannot be applied for the materials under hydrostatic pressure.

12. Write the limitations of Maximum Strain Energy Theory. (Beltrami Theory).

This theory does not apply to brittle materials for which elastic limit in tension and in compression are quite different.

13. Write the failure theories and its relationship between tension and shear.

1. Maximum Principal Stress Theory. (Rakine's theory) $\zeta_y = f_y$

2.Maximum Principal Strain Theory. (St. Venant's theory) $\zeta_y = 0.8\ f_y$

3. Maximum Shear Stress Theory. (Tresca's theory) $\zeta_y = 0.5\ f_y$

4.Maximum Shear Strain Theory (Von– Mises - Hencky theory or Distortion energy theory) $\zeta_y = 0.577\ f_y$

5. Maximum Strain Energy Theory. (Beltrami Theory) $\zeta_y = 0.817 f_y$.

14. Write the volumetric strain per unit volume.

$$f_y^{\ 2} / 2E$$

20. Define : Octahedral Stresses

A plane, which is equally inclined to the three axes of reference, is called octahedral plane. The normal and shearing stress acting on this plane are called octahedral stresses.

$$\tau_{oct} = 1/3 \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

21. Define: Plasticity ellipse.

The graphical surface of a Maximum Shear Strain Theory (Von –Mises- Hencky theory or Distortion energy theory) is a straight circular cylinder. The equation in 2D is

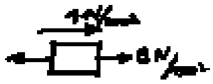
$$\sigma_1^{\ 2} - \sigma_1\sigma_2 + \sigma_2^{\ 2} = f_y^{\ 2}$$ which is called the Plasticity ellipse

PART- B

1 i) Briefly explain spherical and deviatory components of stress tensor.

ii) Explain the importance of theories of failure.

iii) For the state of stress shown in Fig, find the principal plane and principal stress.



2. A circular shaft has to take a bending moment of 9000 N/m and torque 6750 N/m. The stress at elastic limit of the material is 207 × 10^6 N/m^2 both in tension and compression. E = 207 x 10^6 KPa and μ = 0.25.Determine the diameter of the shaft, using octahedral shear stress theory and the maximum shear stress theory. Factor of safety : 2.

(AUC Apr/May 2010)

3. i) State Maximum Shear Stress Theory

ii) A shaft is subjected to a maximum torque of I0 kNm and a maximum of bending moment of 8kNm at a particular section. If the allowable equivalent stress in simple tension is 160MN/m^2, find the diameter of the shaft according to the maximum shear stress theory. **(AUC Nov/Dec 2010)**

4. In a steel member, at a point the major principal stress is 200MN/m^2 and the minor principal stress is compressive. If the tensile yield point of the steel is 235MN/m^2, find

The value of the minor principal stress at which yielding will commence according to each of the following criteria of failure

i) Maximum shearing stress.

ii) Maximum total strain energy and

iii) maximum shear strain energy take poission ratio= 0.26

5. The rectangular stress components of a point in three dimensional stress system are defined as $\sigma_x = 20$ MPa , $\sigma_y = -40$ MPa , $\sigma_z = 80$ MPa , $\tau_{xy} = 40$ MPa , $\tau_{yz} = -60$ MPa and $\tau_{zx} = 20$ MPa . Determine the principal stresses at the given point.

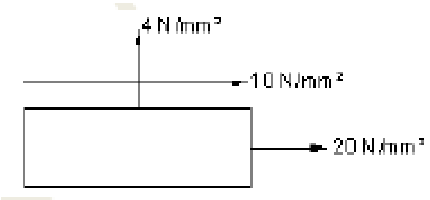
6. A steel shaft is subjected to an end trust producing a stress of 90 Mpa and the max. shearing stress on the surface arising from torsion is 60 MPa the yield point of the material in simple tension was found to be 300 MPa calculate the factor of safety of the shaft according to

i. max. shear stress theory and

ii. max. distortion energy theory.

7. i. State the shear strain energy theory and a comment on it.

ii. for the state of stress shown in fig find the principal plane, principle stress and max. shear stress



8. In a material the principal stresses are 50 N/mm², 40 N/mm² and – 30 N/mm². Calculate the total strain energy, volumetric strain energy, shear strain energy and factor of safety on the total strain energy criterion if the material yields at 100 N/mm².

9. The state of stress at a point is given by the tensor below. Determine the principal stresses and its directions $\begin{bmatrix} 20 & -10 & -30 \\ -10 & 40 & 20 \\ -30 & 20 & -20 \end{bmatrix}$ MPa. **(AUC Apr/May 2012)**

10. Explain any two theories of failure. **(AUC Apr/May 2012)**

11. The normal stress in two mutually perpendicular directions are 600 N/mm² and 300 N/mm² both tensile. The complimentary shear stresses in these directions are of intensity 450 N/mm². Find the normal and tangential stresses in the planes which are equally inclined to the planes carrying the normal stresses mentioned above.

12. A solid circular shaft is subjected to a bending moment of 40 kN m and a torque of 10 kN m. Design the diameter of the shaft according to
    i. Maximum principal stress theory
    ii) Max. shear stress theory
    iii) Max, strain energy theory

UNIT 5 ADVANCED TOPICS IN BENDING OF BEAM

Part- A

1. What are the assumptions made in the analysis of curved bars?
   1.Plane sections remain plane during bending.
   2.The material obeys Hooke's law.
   3.Radial strain is negligible.
   4.The fibres are free to expand or contract without any constraining effect from the adjacent fibres.

2. Write the formula for stress using Winkler-Bach theory?

$$\sigma = \frac{M}{R \times A}\left\{1 + \frac{R^2}{h^2}\left[\frac{y}{R+y}\right]\right\}$$

where σ = Bendind stress (i.e., σ_b)
M = Bending moment with which the bar is subjected
R = Radius of curvature of curved bar or it is the distance of axis of curvature from centroidal axis.
A = Area of cross-section
h^2 = is a constant for a cross-section

$$= \frac{1}{A} \int \frac{y^2 dA}{1 + \left(\frac{y}{R}\right)}$$

3. Define unsymmetrical bending.
If the plane of loading or that of bending, does not lie in (or parallel to) a plane that contains the principal centroidal axisof the cross-section, the bending is called unsymmetrical bending.

4. What are the reasons for unsymmetrical bending?
   1.The section is symmetrical but the load line is inclined to both the principal axes.
   2.The section itself is unsymmetrical and the load line is along the centroidal axis.

5. How will you calculate the stress due to unsymmetrical bending?

$$\sigma = \frac{Mu.u}{Ivv} + \frac{Mv.v}{Iuu}$$

where $u = x \cos\theta + y \sin\theta$
$v = y \cos\theta - x \sin\theta$

6. How will you calculate the distance of neutral axis from centroidal axis.

$$y_0 = -\frac{R \times h^2}{R + h^2}$$

-ve sign shows that neutral axis is below the centroidal axis.

7. How will you calculate the angle of inclination of neutral axis with respect to principal axis?

$$\alpha = \tan^{-1}\left(\frac{I_{UU}}{I_{VV}} \tan\theta\right)$$

8. Write the formula for deflection of a beam causing unsymmetrical bending.

$$\delta = \frac{KWl^3}{E}\sqrt{\frac{\sin^2\theta}{I^2_{vv}} + \frac{\cos^2\theta}{I^2_{uu}}}$$

Where K = a constant depending upon the end conditions of the beam and the position of the load along the beam

l = length of the beam

θ = angle of inclination of load W with respect to VV principal axis

9. How will you calculate the resultant stress in a curved bar subjected to direct stress and bending stress.

$$\sigma_r = \sigma_o + \sigma_b$$

where σ_o = Direct stress = P/A

σ_b = Bending stress

10. How eill you calculate the resultant stress in a chain link.

$$\sigma_r = \sigma_o + \sigma_b$$

where σ_o = Direct stress = $\frac{P}{2A} \times \sin\theta$

σ_b = Bending stress

11. What is shear centre or angle of twist?

The shear centre for any transverse section of the beam is the point of intersection of the bending axis and the plane of the transverse section.

12. Who postulated the theory of curved beam?

Winkler-Bach postulated the theory of curved beam.

13. What is the shape of distribution of bending stress in a curved beam?

The distribution of bending stress is hyperbolic in a curved beam.

14. Where does the neutral axis lie in a curved beam?

The neutral axis does not coincide with the geometric axis.

15. What is the nature of stress in the inside section of a crane hook?

Tensile stress

16. Where does the maximum stress in a ring under tension occur?

The maximum stress in a ring under tension occurs along the line of action of load.

17. What is the most suitable section for a crane?

Trapezoidal section.

18. What is pure bending of a beam?

When the loads pass through the bending axis of a beam, then there shall be pure bending of the beam.

19. How will you determine the product of inertia.

The product of inertia is determined with respect to a set of axes which are perpendicular to each other.

The product of inertia is obtained by multiplying each elementary area dA by its co-ordinates x and y and integrated over the area A.

$I_{XY} = \int xy\, dA$

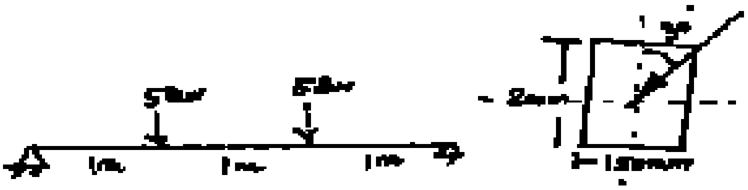
20. Define principal moment of inertia.

The perpendicular axis about which the product of inertia is zero are called "principal axes" and the moments of inertia with respect to these axes are called as principal moments of inertia.

The maximum moment of inertia is known as Major principal moment of inertia and the minimum moment of inertia is known as Minor principal moment of inertia.

PART- B

1. A rectangular simply supported beam is shown in Fig. The plane of loading makes 30° with the vertical plane of symmetry. Find the direction of neutral axis and the bending stress at A. **(AUC Apr/May 2010)**



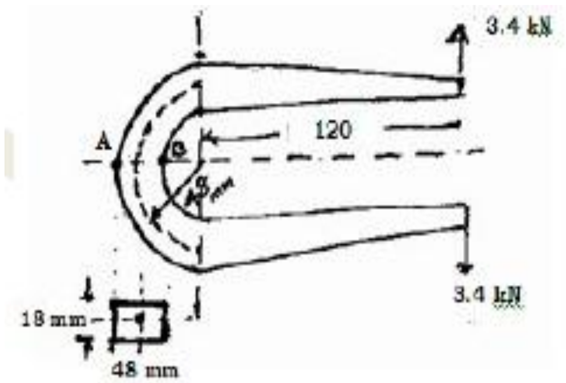
2. A curved bar of rectangular section, initially unstressed is subjected to bending moment of 2000 N.m tends to straighten the bar. The section is 5 cm wide and 6 cm deep in the plane of bending and the mean radius of curvature is 10 m. find the position of neutral axis and the stress at the inner and outer face. **(AUC Apr/May 2010)**
3. A thick cylinder of external and internal diameter of 350 mm and 200 mm is subjected to an internal pressure of 45 N/mm^2 and external pressure $5N/mm^2$. Determine the stress in the material. Now if the external pressure is doubled, what internal pressure can be maintained without exceeding the previously determine maximum stress?

4. Write brief technical note on:
    i) Unsymmetrical bending of beams
    ii) Curved beams
    iii) Stress concentration
    iv) Significance of shear centre.

5. A 80 x 80 x 10 mm angle is used as a simply supported beam over a span of 2.4m. it carries a load of 400kN along the vertical axis passing through the centroid of the section determine the resulting bending stress on the outer corner of the section along the middle section of the beam.

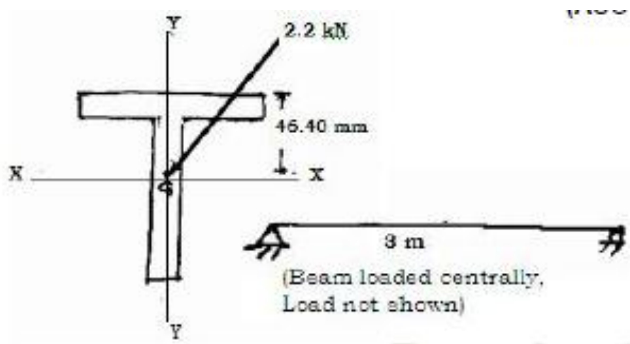
6. A central horizontal section of hook is a symmentrical trapezium 60mm deep the inner width being 60mm and the outer being 30mm. estimate the extrame intensities of stress when the hook carries a load 30kN the load line passing 40mm from the inside edge of the section and the centre of curvature being in the load line

7. fig. shows a frame subjected to a load of 3.4Kn find the resultant stress at A and B

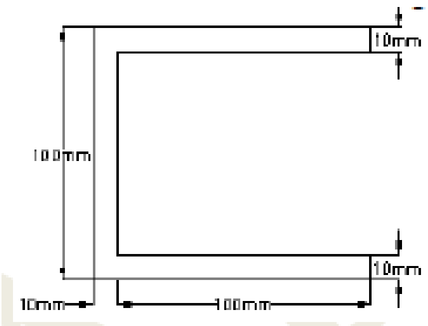


8. A beam of T-section (flange: 100 × 20 mm, web: 150 mm × 10 mm) in 3 m in length and simply supported at ends (Fig). It carries a load of 2.2 kN inclined 20° to the vertical and passing through the centroid of the section. Calculate the maximum tensile stress and maximum compressive stress. Also find the position of the neutral axis.

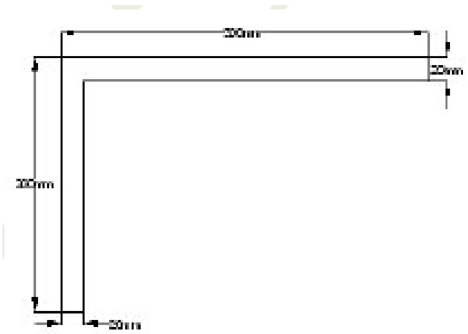
(AUC Nov/Dec 2011)



9. Determine the shear centre for a channel section shown in fig.



10. find the centroidal principal moments of inertia of an angle section 300mm x 200mm x 200mm as shown in fig



11. A curved bar is formed of a tube of 120 mm outside diameter and 7.5 mm thickness. The centre line of this beam is a circular are of radius 225 mm. A bending moment of 3 kN m tending to increase curvature of the bar is applied. Calculate the maximum tensile and compressive stresses set up in the bar.

12. A 80 mm x 80 mm x 10mm angle section shown in fig is used as a simply supported beam over a span 2.4 m. It carries a load of 400 N along the line YG, where G is the centroid of the section. Calculate the

i. Stresses at the points A, B and C of the mid section of the beam
ii. Deflection of the beam at the mid section and its direction with the load line
iii. Position of the neutral axis. Take E = 200 GN/m^2.